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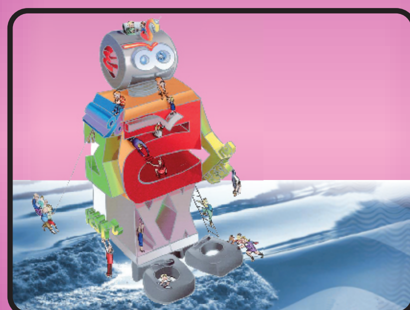
OLYMPIAD EXPLORER



Workbook for

Nationwide Interactive MATHS Olympiad & Other National/International Olympiads/Talent Search Exams.

Based on CBSE, ICSE, GCSE, State Board Syllabus & NCF (NCERT)

100's of Q's with answers

- Chapterwise Practice Q's
- Revision Q's
- Sample Paper

Class 11 & 12

EDUHEAL FOUNDATION

• LEARNING FOR LIFE •

EduHeal Foundation conducts 5 Olympiads annually reaching out to 3,500 + Schools ● 4 Lakh + Students ● 50,000 Coordinating Teachers and having 500 Resource persons in English / Maths / Science / Biotech / Computer & 300 Regional Coordinators.

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Contents

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SYLLABUS GUIDELINES*

Based on CBSE, ICSE & GCSE Syllabus & NCF guidelines devised by NCERT

CLASS - XI

UNIT-I: SETS AND FUNCTIONS

1. Sets:

Sets and their representations. Empty set. Finite & Infinite sets. Equal sets. Subsets. Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams.

Union and Intersection of sets. Difference of sets. Complement of a set.

2. Relations & Functions:

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the reals with itself (upto R R R ´´).

Definition of relation, pictorial diagrams, domain, co-domain and range of a relation.

Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain & range of a function. Real valued function of the real variable, domain and

range of these functions, constant, identity, polynomial, rational, modulus, signum and greatest integer functions with their graphs. Sum, difference, product and quotients of functions.

3. Trigonometric Functions:

Positive and negative angles. Measuring angles in radians & in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the

identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions and sketch of their graphs.

Expressing sin $(x + y)$ and cos $(x + y)$ in terms of sin$x$, siny, cos$x$ & cos$y$. Deducing the identities like

following :

$$\tan(x \pm y) = \frac{\tan x \pm y}{1 \mp \tan x \times \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \pm 1}{\cot x \mp \cot y}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}, \cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}, \cos x = \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Identities related to sin $2x$, cos$2x$, tan $2x$, sin$3x$, cos$3x$ and tan$3x$. General solution of trigonometric equations of the type sin$q$ = sin$a$, cos$q$ = cos$a$ and tan$q$ = tan a. Proofs and simple applications of sine and cosine formulae.

UNIT – II: ALGEBRA

1. Principle of Mathematical Induction:

Processes of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers.

The principle of mathematical induction and simple applications.

2. Complex Numbers and Quadratic Equations:

Need for complex numbers, especially -1 , to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers.

Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system.

3. Linear Inequalities:

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables- graphically.

4. Permutations & Combinations:

Fundamental principle of counting. Factorial n. Permutations and combinations, derivation of formulae and their connections, simple applications.

5. Binomial Theorem:

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications.

6. Sequence and Series:

Sequence and Series. Arithmetic progression (A. P.), arithmetic mean (A.M.). Geometric progression (G.P.), general term of a G. P., sum of n terms of a G.P., geometric mean (G.M.), relation between A.M. and G.M. Sum to n terms of the special series: $\sum n, \sum n^2$ and $\sum n^3$.

UNIT- III: COORDINATE GEOMETRY

1. Straight Lines:

Brief recall of 2D from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two -point form, intercepts form and normal form. General equation of a line. Distance of a point from a line.

2. Conic Sections:

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three -dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point in space. Distance between two points and section formula.

UNIT-IV: CALCULUS

1. Limits and Derivatives:

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive

idea of derivatives. Definition of derivative, limits, limits of trigonometric functions.

UNIT-V: MATHEMATICAL REASONING

1. Mathematical Reasoning:

Mathematically acceptable statements. Connecting words/ phrases - consolidating the understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "61nd", "or", "there exists" and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words-difference between contradiction, converse and contrapositive.

UNIT-VI: STATISTICS & PROBABILITY

1. Statistics:

Measure of dispersion; mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

2. Probability:

Random experiments: outcomes, sample spaces (set representation). Events: occurrence of events, 'not', 'and' & 'or' events, exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability, connections with the theories of earlier classes. Probability of an event, probability of 'not', 'and' & 'or' events.

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SYLLABUS GUIDELINES*

Based on CBSE, ICSE & GCSE Syllabus & NCF guidelines devised by NCERT

CLASS- XII

Compulsory for all

RELATIONS AND FUNCTIONS

1. Relations and Functions:

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions:

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

ALGEBRA

1. Matrices:

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants:

Determinant of a square matrix (upto 3×3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

CALCULUS

1. Continuity and Differentiability:

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivative. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

2. Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents & normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as reallife situations).

3. Integrals:

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{ax^2 + bx + c}$$

$$\int \frac{(px+q)}{ax^2 + bx + c} dx, \int \frac{(px+q)}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx \text{ and } \int \sqrt{x^2 - a^2 dx}$$

to be evaluated.

Definite integrals as a limit of a sum.

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals

4. Applications of the Integrals:

Applications in finding the area under simple curves, especially lines, arcs of circles/ parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

5. Differential Equations:

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type :

$\frac{dy}{dx} + p(x)y = q(x)$, where $p(x)$ and $q(x)$ are functions of x.

LINEAR PROGRAMMING

1. Linear Programming:

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constrains).

PROBABILITY

1. Probability:

Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem. Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

For Science stream students

1. Vectors:

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors.

2. Three - dimensional Geometry:

Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

For Non - Science stream students

1. Partnership

Basic definitions, sharing of profits, partner's salaries and interest on Capital, Profit sharing on Admission of a New Partner of Retirement of an existing partner.

2. Bill of Exchange

Bill of Exchange, True Discount, Banker's Discount and Banker's Gain.

3. Linear Programming

Linear Programming Problems, Different Areas of Applications of Linear Programming Problems, Basic Concepts of Linear Programming Problems, Mathematical Formulation of a Linear Programming Problem, Advantages of Linear Programming Problems, Limitations of Linear Programming, The Graphical Method of Solving and LPP, Some Exceptional Cases.

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CHAPTER 1

Matrices and Determinants

Q.1. If A and B are square matrices of equal degree, then which one is correct among the followings?

(a) $A + B = B + A$ (b) $A + B = A - B$

(c) $A - B = B - A$ (d) $AB = BA$.

Q.2. The parameter, on which the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

does not depend upon

(a) a (b) p (c) d (d) x.

Q.3. The determinant

$$\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0, \text{ if}$$

(a) x, y, z are in A.P. (b) x, y, z are in G.P.

(c) x, y, z are in H.P. (d) xy, yz, zx are in A.p.

Q.4. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$

(c) $x = 0, y = 3$ (d) $x = 0, y = 0$.

Q.5. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ equals

(a) 0 (b) 1 (c) 100 (d) –100

Q.6. If the system of equations $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solutions, then possible value of k are

(a) –1, 2 (b) 1, 2 (c) 0, 1 (d) –1, 1.

Q.7. The no. of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) 3

Q.8. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$, is

(a) 1 (b) –1
(c) 4 (d) no real values

Q.9. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

(a) –1 (b) 1
(c) 0 (d) no real values.

Q.10. Given $2x - y - z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ then the value of λ such that the given system of equation has no solution, is

(a) 3 (b) –2 (c) 0 (d) –3.

Q.11. The determinant $\begin{vmatrix} 1 & (x-3) & (x-3)^2 \\ 1 & (x-4) & (x-4)^2 \\ 1 & (x-5) & (x-5)^2 \end{vmatrix}$ vanishes for :

(a) 3 values of x (b) 2 values of x
(c) 1 value of x (d) no value of x

Q.12. If $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x^2 \\ 1 & x^2 & x^3 \end{vmatrix}$, then $f'(x)$ vanishes at :

(a) $x = -1$ (b) $x = 0$ (c) $x = 1$ (d) none of these

Q.13. If $a + b + c = 0$, then one root of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 \text{ is :}$$

(a) $x = 1$ (b) $x = 2$
(c) $x = a^2 + b^2 + c^2$ (d) $x = 0$

Q.14. The value of the determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$ is equal to :

(a) x (b) y (c) z (d) 0

Q.15. Consider the system of linear equations

$$a_1x + b_1y + c_1z + d_1 = 0, \quad a_2x + b_2y + c_2z + d_2 = 0$$

and $a_3x + b_3y + c_3z + d_3 = 0$.

Let us denote by $\Delta(a, b, c)$ the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$,

if $\Delta(a, b, c) \ne 0$, then the value of x in the unique solution of the above equations is :

(a) $\frac{\Delta(b,c,d)}{\Delta(a,b,c)}$ (b) $\frac{-\Delta(d,b,c)}{\Delta(a,b,c)}$ (c) $\frac{\Delta(a,c,d)}{\Delta(a,b,c)}$ (d) $-\frac{\Delta(a,b,d)}{\Delta(a,b,c)}$

Q.16. If $a + b + c = 0$, then determinant

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \text{ is equal to :}$$

(a) 0 (b) 1 (c) 2 (d) 3

Q.17. The value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix} \text{ is zero, if :}$$

(a) $\sin x = 0$ (b) $\cos x = 0$

(c) $a = 0$ (d) $\cos x = \frac{1+a^2}{2a}$

Q.18. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$, then the value of $\frac{p}{x} + \frac{q}{y} + \frac{r}{z}$ is :

(a) 0 (b) 1 (c) 2 (d) $4pqr$

Q.19. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x)$ equals to :

(a) $a(2a + 3x)$ (b) $ax(2x + 3a)$
(c) $ax(2a + 3x)$ (d) $x(2a + 3x)$

Q.20. The determinant $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix}$ is equal to zero, if a, b, c are in :

(a) GP (b) AP (c) HP (d) none of these

Q.21. If $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$, then adj (A) is equal to :

(a) $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -6 \\ 5 & -7 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -5 \\ 6 & -2 \end{bmatrix}$ (d) none of these

Q.22. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is –ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

(a) +ve (b) $(ac - b^2)(ax^2 + 2bx + c)$
(c) –ve (d) 0.

Q.23. l, m, n are the p^{th}, q^{th} and r^{th} term of a GP, all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) –1 (b) 2 (c) 1 (d) 0.

Q.24. If 1, ω, ω^2 are the cube roots of unity, then

$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0.

Q.25. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2, \beta = 2ab$ (b) $\alpha = 2ab, \beta = a^2 + b^2$
(c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (d) $\alpha = a^2 + b^2, \beta = ab$.

Q.26. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals

(a) –1 (b) 1 (c) 0 (d) 2.

Q.27. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is

(a) A^{-1} does not exist
(b) $A = (-1) I$, where I is a unit matrix
(c) A is a zero matrix (d) $A^2 = 1$.

Q.28. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and 10 B = $\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

(a) 2 (b) –1 (c) –2 (d) 5.

Q.29. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in G.P., then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is

(a) 2 (b) 1 (c) 0 (d) –2.

Q.30. If $A^2 - A + I = 0$, then the inverse of A is

(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$

Q.31. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all n ≥ 1, by the principle of mathematical induction

(a) $A^n = 2^{n-1} A - (n-1)I$ (b) $A^n = nA - (n-1)I$
(c) $A^n = 2^{n-1} A + (n-1)I$ (d) $A^n = nA + (n-1)I$.

Q.32. If $a^2 + b^2 + c^2 = -2$ and

$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$

then $f(x)$ is a polynomial of degree

(a) 0 (b) 1 (c) 2 (d) 3.

Q.33. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solutions, if α is

(a) either –2 or 1 (b) –2
(c) 1 (d) not – 2.

Q.34. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?

(a) $A = B$ (b) $AB = BA$

(c) either A or B is a zero matrix
(d) either A or B is an identity matrix

Q.35. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a, b $\in$ N. Then
(a) there cannot exit any B such that $AB = BA$
(b) there exist more than one but finite number B's such that $AB = BA$
(c) there exists exactly one B such that $AB = BA$
(d) there exist infinitely many B's such that $AB = BA$

Q.36. If for $x \neq 0, y \neq 0$ then D is
(a) Divisible by x but not y (b) Divisible by y but not x
(c) Divisible by neither x nor y (d) Divisible by both x and y

Q.37. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

If $|A^2| = 25$, then $|\alpha|$ equals
(a) 1/5 (b) 5 (c) 5^2 (d) 1

Q.38. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?
(a) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers
(b) If det $A = \pm 1$, then A^{-1} need not exist
(c) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(d) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers

Q.39. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.
Statement-1: If $A \neq I$ and $A \neq -I$, then det A = -1.
Statement-2: If $A \neq I$ and $A \neq -I$, then tr$(A) \neq 0$.
(a) Statement-1 is true, statement-2 is true; statement -2 is not a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is false.
(c) Statement-1 is false, statement-2 is true.
(d) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

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ANSWERS

1. (a)	2. (b)	3. (b)	4. (d)	5. (a)	6. (d)	7. (c)	8. (d)
9. (a)	10. (b)	11. (d)	12. (c)	13. (d)	14. (d)	15. (b)	16. (a)
17. (a)	18. (c)	19. (c)	20. (a)	21. (a)	22. (c)	23. (d)	24. (d)
25. (a)	26. (a)	27. (d)	28. (d)	29. (c)	30. (c)	31. (d)	32. (c)
33. (b)	34. (b)	35. (d)	36. (d)	37. (a)	38. (a)	39. (b)	

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CHAPTER 2

Complex Numbers

Q.1. The inequality $|z - 4| < |z - 2|$ represents the region given by
(a) Re $(z) > 0$ (b) Re $(z) < 2$
(c) Re $(z) < 0$ (d) none of these

Q.2. If $z = x + iy$ and $w = (1 - iz) / (z - i)$, then $|w| = 1$ implies that, in the complex plane:
(a) z lies on the imaginary axis
(b) z lies on the real axis
(c) z lies on the unit circle (d) none of these

Q.3. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order, if and only if :
(a) $z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$
(c) $z_1 + z_2 = z_3 + z_4$ (d) none of these

Q.4. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1 - r)\, a + rb$ and $w = (1 - r)\, u + rv$, where r is a complex number, then the two triangles:
(a) have the same area (b) are similar
(c) are congruent (d) none of these

Q.5. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is:
(a) -1 (b) 0 (c) $-i$ (d) i

Q.6. If z_1 and z_2 are two non zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to :
(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$

Q.7. The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, for :
(a) $x = n\pi$ (b) $x = 0$
(c) $x = (n + {}^1/_2)\,\pi$ (d) no value of x

Q.8. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19}, β^7 is
(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

Q.9. If ω is an imaginary cube root of unity, then the value of $\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\}$ is

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Q.10. If ω $(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and B are respectively :
(a) 0,1 (b) 1,1 (c) 1, 0 (d) –1, 1

Q.11. If ω is a complex cube root of unity, then the value of $\omega^{99} + \omega^{100} + \omega^{101}$ is :
(a) 1 (b) –1 (c) 3 (d) 0

Q.12. Real part of $\frac{1}{1-\cos\theta + i\sin\theta}$ is equal to :
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}\tan\frac{\theta}{2}$ (d) 2

Q.13. If $z = x + iy$ and $\omega = \frac{1-iz}{z-i}$, then $|\omega| = 1$ implies that in the complex plane :
(a) z lies on the imaginary axis
(b) z lies on the real axis
(c) z lies on the unit circle (d) none of these

Q.14. Let $z = 1 - t + i\sqrt{t^2+t+2}$, where t is a real parameter. The locus of z in the argand plane is :
(a) an ellipse (b) a hyperbola
(c) a straight line (d) none of these

Q.15. If 1, ω, ω^2, ..., ω^{n-1} are the n roots of unity, then : $(1 - \omega)(1 - \omega^2) \ldots (1 - \omega^{n-1})$ equals :
(a) 0 (b) 1 (c) n (d) n^2

Q.16. If $(1 + i)(1 - 2i)(1 - 3i) \ldots (1 - ni) = \alpha - i\beta$, then $\alpha^2 + \beta^2$ equals
(a) $1\cdot2\cdot3 \ldots n$ (b) $1^2\cdot2^2\cdot3^2 \ldots n^2$
(c) $1^2 + 2^2 + 3^2 + \ldots n^2$ (d) $2\cdot5\cdot10 \ldots (n^2 + 1)$

Q.17. If $z = (\lambda + 3) + i\sqrt{5-\lambda^2}$, then the locus of z is a:
(a) circle (b) hyperbola
(c) parabola (d) none of these

Q.18. If ω is a non-real cube root of unity, then
$\frac{1+2\omega+3\omega^2}{2+3\omega+\omega^2} + \frac{2+3\omega+3\omega^2}{3+3\omega+2\omega^2}$ is equal to :
(a) -2ω (b) 2ω (c) ω (d) 0

Q.19. The value of
$(1 + \omega^2 + 2\omega)^{3n} - (1 + \omega + 2\omega^2)^{3n}$ is :
(a) zero (b) 1 (c) ω (d) ω^2

Q.20. The value of $i^{1/3}$ is :
(a) $\frac{\sqrt{3}+i}{2}$ (b) $\frac{\sqrt{3}-i}{2}$ (c) $\frac{1+i\sqrt{3}}{2}$ (d) $\frac{1-i\sqrt{3}}{2}$

Q.21. Least value of n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n$ is an integer, is :
(a) 1 (b) 2 (c) 3 (d) 4

Q.22. z and w are two non-zero complex number such that $|z| = |w|$ and Arg z + Arg $w = \pi$ then z equals
(a) $\overline{w}$ (b) $-\overline{w}$ (c) w (d) $-w$.

Q.23. If $|z-4| < |z-2|$, its solution is given by
(a) Re(z) > 0 (b) Re(z) < 0
(c) Re(z) > 3 (d) Re(z) > 2.

Q.24. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

Q.25. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then
(a) $x = 2n$, where n is any positive integer
(b) $x = 4n + 1$, where n is any positive integer
(c) $x = 2n + 1$, where n is any positive integer
(d) $x = 4n$, where n is any positive integer.

Q.26. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and Arg(z) – Arg(ω) = $\pi/2$, then $\bar{z}\omega$ is equal to
(a) –1 (b) i (c) $-i$ (d) 1.

Q.27. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex further, assume that the origin, z_1 and z_2 form an equilateral triangle, then
(a) $a^2 = 2b$ (b) $a^2 = 3b$ (c) $a^2 = 4b$ (d) $a^2 = b$.

Q.28. Let z, w be complex numbers such that $z + \overline{iw} = 0$ and $zw = \pi$. Then arg z equals
(a) $3\pi/4$ (b) $\pi/2$ (c) $\pi/4$ (d) $5\pi/4$.

Q.29. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to

(a) 2 (b) –1 (b) 1 (d) –2.

Q.30. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(a) a circle (b) the imaginary axis
(c) the real axis (d) an ellipse.

Q.31. If z_1 and z_2 are two non – zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to
(a) $-\pi$ (b) $\pi/2$ (c) $-\pi/2$ (d) 0

Q.32. If $\omega = \dfrac{z}{z-(1/3)i}$ = and $|\omega| = 1$, then z lies on
(a) a circle (b) an ellipse
(c) a parabola (d) a straight line.

Q.33. The value of $\sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right)$ is

(a) i (b) 1 (c) –1 (d) $-i$

Q.34. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots + \left(z^6+\frac{1}{z^6}\right)^2$$

(a) 18 (b) 54 (c) 6 (d) 12

Q.35. If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is
(a) 6 (b) 0 (c) 4 (d) 10

Q.36. The conjugate of a complex number is $\dfrac{1}{i-1}$. Then that complex number is
(a) $\dfrac{-1}{i+1}$ (b) $\dfrac{1}{i-1}$ (c) $\dfrac{-1}{i-1}$ (d) $\dfrac{1}{i+1}$.

☺☺☺

ANSWERS

1. (d)	2. (b)	3. (b)	4. (b)	5. (d)	6. (c)	7. (d)	8. (d)
9. (c)	10. (b)	11. (d)	12. (b)	13. (b)	14. (b)	15. (c)	16. (d)
17. (a)	18. (b)	19. (a)	20. (a)	21. (c)	22. (b)	23. (c)	24. (b)
25. (d)	26. (c)	27. (b)	28. (a)	29. (d)	30. (d)	31. (d)	32 (d)
33. (d)	34. (d)	35. (a)	36. (a)				

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NATIONWIDE INTERACTIVE MATHS OLYMPIAD (NIMO)SAMPLE PAPER

Total duration : 60 Minutes **Total Marks :** 50

CLASS - 11

SECTION - A

MENTAL ABILITY

1. Given Set : (256, 64, 16), Choose the similar set?

(a) (160, 40, 10) (b) (64, 32, 8)

(c) (144, 36, 9) (d) None of these

2. If $Z = 52$ and ACT = 48, then BAT will be equal to

(a) 39 (b) 41 (c) 46 (d) None of these

Direction : *Q. (3-4) In certain code language*

(A) '*pic vic nic*' means '*winter is cold*'.

(B) '*to nic re*' means '*summer is hot*'

(C) '*re pic boo*' means '*winter and summer*';

(D) '*vic tho pa*' means '*nights are cold*'.

3. Which word in that language means '*summer'?*

(a) nic (b) re (c) to (d) None of these

4. Which of the given statements is superfluous?

(a) Only A (b) Only D

(c) Both A and D (d) Neither A not D

Direction : *(Q. 5) Read the following information carefully and answer the question based on it:*

A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not mother of Q. P and R are a married couple. Y is the brother of R. X is the daughter of P. Z is the brother of P.

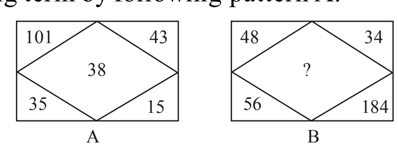
5. Who is the brother in-law of R ?

(a) P (b) Z (c) Y (d) None of these

6. I start from my home and go 2 km straight. Then, I turn towards my right and go 1 km. I turn again towards my right and go 1 km again. If I am north-west from my house, then in which direction did I go in the begining?

(a) North (b) South (c) West (d) None of these

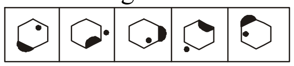
7. Find the missing term by following pattern A.



(a) 127 (b) 142 (c) 158 (d) None of these

Direction : *In the following question consists of five figure marked A, B, C, D and E called the Problem Figures followed by three other figures marked a, b, and c called the Answer Figures. Select a figure from amongst the Answer Figures which will continue the same series as established by the five problem Figures.*

8. Problem figures



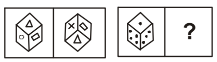
A B C D E

Answer figures

(a)  (b)  (c)  (d) None of these

Direction : *Given question consist of two sets of figures A, B, C and D constitute the problem set while figures (a), (b) and (c) constitute the answer set. There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by choosing a suitable figure (D) from the answer set.*

9. Problem set



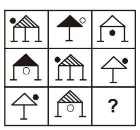
A B C D

Answer set

(a)  (b)  (c)  (d) None of these

Direction : *In the following questions, find out which of the answer figures (a), (b) and (c) completes the figure-matrix?*

10.



(a)  (b)  (c)  (d)None of these

SECTION - B

MATHEMATICS

11. Let R be a relation defined as $R = \{(a, b) : a \le b)\}$ where a, b are real numbers. Then relation R is
(a) Reflexive, symmetric and transitive
(b) Reflexive and transitive but not symmetric
(c) Symmetric and transitive but not reflexive
(d) None of these

12. $\dfrac{1+i^2+i^3+i^4+i^5}{1+i}$ equals
(a) $1 + i$ (b) $(1+ i)/2$ (c) $(1 – i)/2$ (d) None of these

13. The roots of the equation $(x – 1)^3 + 8 = 0$ are
(a) $1 + 2\omega, 1 + 2\omega^2, – 1$ (b) $1 – 2\omega, 1 – 2\omega^2, – 1$
(c) $1 – 2\omega, 1 + 2\omega^2, – 1$ (d) None of these

14. If α,β are roots of the equation $x^2 + x + 1 = 0$ and α/β, β/α are roots of the equation $x^2 + px + q = 0$, then p equals :
(a) –1 (b) 1 (c) –2 (d) None of these

15. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in
(a) AP (b) GP (c) HP (d) None of these

16. The sum of all numbers between 100 and 10,000 which are of the form n^3 $(n \in N)$ is equal to
(a) 55216 (b) 53261 (c) 51261 (d)None of these

17. $\log_e n + \dfrac{(\log_e n)^3}{3!} + \dfrac{(\log_e n)^5}{5!} + ...$ upto ∞ is equal to
(a) $\dfrac{n^2-1}{2n}$ (b) $\dfrac{n^2+1}{2n}$ (c) $\dfrac{n(n-1)}{2n}$ (d)None of these

18. Every body in a room shakes hands with everybody else. If total number of hand-shaken is 66, then total number of persons in the room is
(a) 11 (b) 12 (c) 13 (d)None of these

19. The sum of the series $1+\dfrac{1}{4}+\dfrac{1.3}{4.8}+\dfrac{1.3.5}{4.8.12}+.....$ is equal to
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $2^{1/3}$ (d) None of these

20. $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix}$ is equal to
(a) 1 (b) 0 (c) $9ab$ (d) None of these

21. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 – 4A – n\text{I} = 0$, then n is equal to
(a) 3 (b) –3 (c) 1/3 (d) None of these

22. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector, then
(a) $3a^2 + 2ab + 3b^2 = 0$ (b) $3a^2 + 10\,ab + 3b^2 = 0$
(c) $3a^2 – 2ab + 3b^2 = 0$ (d) None of these

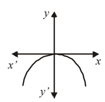
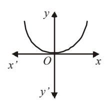
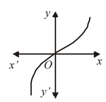
23. A circle passes through (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally. The locus of its centre is

(a) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(b) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(d) None of these

24. The eccentricity of the hyperbola $4x^2 - 9y^2 - 8x = 32$ is
(a) $\sqrt{5}/3$ (b) $\sqrt{13}/3$ (c) $\sqrt{13}/2$ (d) None of these

25. A line makes 45° angle with positive x-axis and makes equal angles with positive *y, z* axes respectively. The sum of the three angles which the line makes with positive *x, y, z* axes is
(a) 180° (b) 165° (c) 150° (d) None of these

26. A line with directors cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points of intersection are given by
(a) $(3a, 2a, 3a)$ $(a, a, 2a)$ (b) $(3a, 2a, 3a)$, (a, a, a)
(c) $(3a, 3a, 3a)$, (a, a, a) (d) None of these

27. Which one of the following represents a graph of even function?
(a)  (b)  (c)  (d) None of these

28. $\lim_{x \to 0} \dfrac{[\sin(x+a) + \sin(a-x) - 2\sin a]}{x \sin x}$ is equal to
(a) $\sin a$ (b) $-\sin a$ (c) 1 (d) None of these

29. If $f(x) = \cos^{-1}\left(\sin\sqrt{\dfrac{1+x}{2}}\right) + x^x$, then at $x = 1$, $f'(x)$ is equal to
(a) 3/4 (b) 1/2 (c) –1/2 (d) None of these

30. If the line $\dfrac{x}{a} + \dfrac{y}{b} = 2$ touches the curve $\left(\dfrac{x}{a}\right)^n + \left(\dfrac{y}{b}\right)^n = 2$ at point (a, b), then n is equal to
(a) 1 (b) 2 (c) 3 (d) None of these

31. A function is matched against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	Function
(a)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$
(b)	$(-\infty, 1/3)$	$3x^2 - 2x + 1$
(c)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(d)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$

32. The median of 19 observations is 30. Two more observations are made and the values of these are 8 and 32. The median of the 21 observations taken together is equal to
(a) 28 (b) 30 (c) 32 (d) None of these

33. If $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, is the set of n observations whose mean is $\bar{x}$ then
(a) $\sum_{i=1}^{n} x_i - \bar{x} \ge 0$ (b) $\sum_{i=1}^{n} (x - \bar{x}_i) = 0$
(c) $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$ (d) None of these

34. The curve represented by differential equation $(1 + y^2)\,dx - xy\,dy = 0$ and passing through point (1, 0) will be
(a) a circle (b) a parabola
(c) an ellipse (d) None of these

35. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other; then angle between $\vec{a}$ and $\vec{b}$ is
(a) 45° (b) 60°
(c) $\cos^{-1}(1/3)$ (d) None of these

36. Two dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is
(a) 1/9 (b) 1/3 (c) 1/4 (d) None of these

37. Let A and B be two events such that $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$ and $P(\overline{A}) = 1/4$. Then events A and B are
(a) Mutually exclusive and independent
(b) Independent but not equally likely
(c) Equally likely but not independent
(d) None of these

38. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ is equal to
(a) 2 (b) 0 (c) 4 (d) None of these

39. In triangle ABC if a, b, c are in AP, then $\tan\dfrac{A}{2}$, $\tan\dfrac{B}{2}$, $\tan\dfrac{C}{2}$ will be in
(a) AP (b) GP (c) HP (d) None of these

40. A balloon is observed simultaneously from three points. A, B and C on a straight road directly under it. The angular elevation at B is twice and at C is thrice that of A. If the distance between A and B is 200 metres and distance between B and C is 100 metres, then the height of the balloon is
(a) 50 m (b) $50\sqrt{2}$ m (c) $100\sqrt{3}$ m (d) None of these

41. $\tan\left(\dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right) + \tan\left(\dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}\dfrac{a}{b}\right)$ is equal to
(a) $2b/a$ (b) b/a (c) $2a/b$ (d) None of these

42. If $n \in N$ and $n > 1$, then

(a) $\lfloor\underline{n} > \left(\frac{n+1}{2}\right)^n$ (b) $\lfloor\underline{n} \geq \left(\frac{n+1}{2}\right)^n$

(c) $\lfloor\underline{n} < \left(\frac{n+1}{2}\right)^n$ (d) None of these

43. If O is the origin and $A(x_1, y_1)$; $B(x_2, y_2)$ are two points, then $OA.OB \cos\angle AOB$ equals

(a) $x_1y_2 + x_2y_1$ (b) $x_1x_2 + y_1y_2$

(c) $x_1y_2 - x_2y_1$ (d) None of these

44. Let a_1, a_2, a_3 .....be terms of an AP. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}, p \neq q,$ then $\frac{a_6}{a_{21}}$ equals

(a) 2/7 (b) 41/11 (c) 11/41 (d) None of these

SECTION - C

INTERACTIVE SECTION

COMPREHENSION - 1

Let consider quadratic equation

$ax^2 + bx + c = 0$ ...(i)

where $a, b, c \in R$ and $a \neq 0$. If eq. (i) has roots, α, β

$\therefore \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$ and eq. (i) can be written as

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

Also, if a_1, a_2, a_3, a_4, ... are AP, then $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 =$ ...... $\neq 0$ and if b_1, b_2, b_3, b_4.....are in GP, then $\frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = ... \neq 1$

Now, c_1, c_2, c_3, c_4.....are in HP, then

$\frac{1}{c_2} - \frac{1}{c_1} = \frac{1}{c_3} - \frac{1}{c_2} = \frac{1}{c_4} - \frac{1}{c_3} = ... \neq 0$

On the basis of above information, answer the following questions:

45. Let p and q be roots of the equation $x^2 - 2x + A = 0$ and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression. Then the values of A and B respectively are

(a) –5, 67 (b) –3, 77 (c) 77, –3 (d) None of these

46. Let α_1, α_2 be the roots of $x^2 - x + p = 0$ and α_3, α_4 be the roots of $x^2 - 4x + q = 0$. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are in GP, then the integral values of p and q respectively are

(a) –2, –32 (b) –2, 3 (c) –6, 32 (d) None of these

47. If a, b, c, d and x are distinct real numbers such that $(a^2 + b^2 + c^2)x^2 - 2(ab+bc+cd)x + (b^2 + c^2 + d^2) \leq 0$, then a, b, c, d

(a) are in AP (b) are in GP

(c) satisfy $ab = cd$. (d) None of these

COMPREHENSION - 2

If $P_n = \sin^n\theta + \cos^n\theta$ where $n \in W$ (whole number) and $\theta \in$ (real number)

On the basis of above information, answer the following questions:

48. If $P_1 = m$, then the value of $4(1 - P_6)$ is

(a) $3(m - 1)^2$ (b) $3(m^2 - 1)^2$

(c) $3(m + 1)^2$ (d) None of these

49. The value of $2P_6 - 3P_4 + 10$ is

(a) 0 (b) 6 (c) 9 (d) None of these

50. The value of $\frac{P_7 - P_5}{P_5 - P_3}$ is

(a) $\frac{P_7}{P_5}$ (b) $\frac{P_5}{P_3}$ (c) $\frac{P_3}{P_1}$ (d) None of these

☺ END OF THE EXAM ☺

ANSWERS

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (b)
6. (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (a)
16. (b)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (b)
26. (c)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (d)
31. (b)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (b)
36. (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (c)
41. (a)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (b)
46. (a)	**47.** (b)	**48.** (b)	**49.** (c)	**50.** (c)

☺☺☺

NATIONWIDE INTERACTIVE MATHS OLYMPIAD (NIMO)SAMPLE PAPER

Total duration : 60 Minutes **Total Marks :** 50

CLASS - 12

SECTION - A

MENTAL ABILITY

1. *Choose the correct alternative that will continue the same pattern.*

$\frac{2}{\sqrt{5}}, \frac{3}{5}, \frac{4}{5\sqrt{5}}, \frac{5}{25}, (.....)$

(a) $\frac{6}{5\sqrt{5}}$ (b) $\frac{6}{25\sqrt{5}}$ (c) $\frac{7}{125}$ (d) None of these

2. If MOBILITY is coded as 46293927, then EXAMINATION is coded as

(a) 45038401854 (b) 56149512965
(c) 57159413955 (d) None of these

3. In a certain code language, '*kew xas huma deko*' means '*she is eating apples*', '*kew tepo qua*' means '*she sells toys*' and '*sul lim deko*' means '*I like apples*'. Which word in that language means '*she*' and '*apples*'?

(a) xas & deko (b) xas & kew
(c) kew & deko (d) None of these

***Directions :** Read the information given below and answer the question.*

I. A, B, C, D, E and F are six members of a family?
II. One couple has parents and their children in the family.
III. A is the son of C and E is the daughter of A.
IV. D is the daughter of F who is the mother of E.

4. How many female members are there in the family?

(a) Two (b) Three (c) Four (d) None of these

***Direction:** Study the information given below and answer the question that follow :*

$A + B$ means A is the daughter of B; $A - B$ means A is the husband of B; $A \times B$ means A is the brother of B.

5. If $P + Q - R$, then which of the following is true?

(a) R is the mother of P (b) R is the sister-in-law of P
(c) R is the aunt of P (d) None of these

6. Two buses start from the opposite points of a main road, 150 kms apart. The first bus runs for 25 kms and takes a right turn and then runs for 15 kms. It then turns left and runs for another 25 kms and takes the direction back to reach the main road. In the meantime, due to a minor breakdown, the other bus has run only 35 kms along the main road. What would be the distance between the two buses at this point?

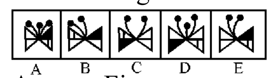
(a) 65 kms (b) 75 kms (c) 80 kms (d) None of these

7. You are alone in the house and there is quite a danger of thieves around. Just then, you hear a knock at the door. You would:

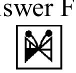
(a) open the door to see who is there.
(b) first peep out from the window to confirm whether you know the person.
(c) not open the door.
(d) ask the servant to see who is there.

***Direction :** In the following question consists of five figure marked A, B, C, D and E called the Problem Figures followed by three other figures marked a, b, and c called the Answer Figures. Select a figure from amongst the Answer Figures which will continue the same series as established by the five problem Figures.*

8. Problem Figures

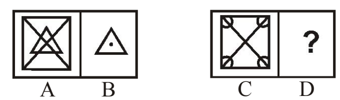


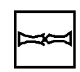
Answer Figures

(a)  (b)  (c)  (d) None of these

***Direction :** In the given question, there is some relationship between the figures A and B. The same relationship exists between the figure C and one of the three alternatives (a), (b), and (c). Choose that figure alternative.*

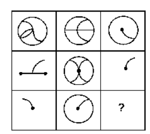
9.

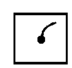
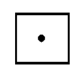


(a)  (b)  (c)  (d) None of these

***Directions :** In the following question find out which of the answer figures (a), (b)and (c) completes the figure - matrix?*

10.



(a)  (b)  (c)  (d) None of these

SECTION - B

MATHEMATICS

11. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 2)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. Then relation R is

(a) not symmetric (b) transitive
(c) a function (d) None of these

12. Match list I (Differential Equation) with List II (Its Solution) and select the correct answer using the codes given below the Lists:

List I (Differential Equation)	List II (Its Solution)
A. $yy' = sec^2 x$	1. $y \sec^2 x = \sec x + c$
B. $y' = x \sec y$	2. $xy = \cos y + c$
C. $y' + (2 \tan x) y = \sin x$	3. $xy = \sin x + c$
D. $xy' + y = \cos x$	4. $y^2 = 2 \tan x + c$
	5. $x^2 = 2 \sin y + c$

Codes :

	A	B	C	D
(a)	3	2	5	4
(b)	4	1	2	3
(c)	4	5	1	3

(d) None of these

13. $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x - \alpha) + C$, then value of (A, B) is
(a) $(-\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(\sin\alpha, \cos\alpha)$ (d) None of these

14. The first three of four given numbers are in GP and their last three are in AP with common difference 6. If first and fourth numbers are equal then the first number is
(a) 2 (b) 8 (c) 6 (d) None of these

15. $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+.....\right)^2$ is equal to
(a) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+..........$ (b) $1+\frac{2x^2}{2!}+\frac{2^3x^4}{4!}+\frac{2^4x^5}{6!}+......$
(c) $1+\frac{2^2x^2}{2!}+\frac{2^4x^4}{4!}+.....$ (d) None of these

16. A student is to answer 10 out of 13 questions in an examination such that he must choose atleast 4 from the first 5 questions. The number of choices available to him is
(a) 346 (b) 140 (c) 196 (d) None of these

17. In the expansion of $(1 + x)^n (1 + y)^n (1 + z)^n$, the sum of the coefficients of the terms of degree m is
(a) ${}^nC_{3m}$ (b) ${}^{3n}C_m$ (c) $({}^nC_m)^3$ (d) None of these

18. If A, B, C are angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$$

then ΔABC is
(a) isosceles (b) equilateral
(c) right-angled (d) None of these

19. If the orthocentre and centroid of a triangle are (–3, 5) and (3, 3) respectively, then its circumcentre is
(a) (6, 2) (b) (6, –2) (c) (0, 4) (d) None of these

20. The angle between lines represented by $x^2 + 4xy + y^2 = 0$ is
(a) 15° (b) 60° (c) 45° (d) None of these

21. If circles $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other, then
(a) $g_1 + g_2 = f_1 + f_2$ (b) $g_1g_2 = f_1f_2$
(c) $f_1g_2 = f_2g_1$ (d) None of these

22. The point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is
(a) (– 9/8, 9/2) (b) (2, – 4)
(c) (9/8, 9/2) (d) None of these

23. The conic represented by $x = 2 (\cos t + \sin t)$, $y = 5 (\cos t - \sin t)$ is
(a) a circle (b) a parabola
(c) an ellipse (d) None of these

24. The point where the line joining points (2, –3, 1) and (3, – 4, –5) meets the plane $2x + y + z = 7$ will be
(a) (1, 2, 7) (b) (–1, 2, 7)
(c) (1, –2, 7) (d) None of these

25. The sum of the direction cosines of a line which makes equal angles with the positive direction of coordinate axes is
(a) 3 (b) $3/\sqrt{2}$ (c) $\sqrt{3}$ (d) None of these

26. The equation of the plane through the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ and parallel to the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ is
(a) $x - 2y - z + 7 = 0$ (b) $x - 2y + z - 7 = 0$
(c) $x - 2y + z + 7 = 0$ (d) None of these

27. If $f(x) = |x|$ and $g(x) = [x]$, then $(fog)(-1/2) + (gof)(-1/2)$ equals
(a) 0 (b) 1 (c) –1 (d) None of these

28. The domain of the function $y(x)$ defined by $2^x + 2^y = 2$ is
(a) $-\infty < x < 1$ (b) $0 \le x \le 1$
(c) $-\infty < x \le 0$ (d) None of these

29. $\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}$ equals
(a) 1 (b) –1 (c) 1/2 (d) None of these

30. If $f(x)=\begin{cases}\dfrac{x^3+x^2-16x+20}{(x-2)^2}, & x\neq 2\\ k, & x=2\end{cases}$ is continuous for all values of x, then the value of k is

(a) 5 (b) 6 (c) 7 (d) None of these

31. If $y=\log\sqrt{\dfrac{1-\sin x}{1+\sin x}}$, then $\dfrac{dy}{dx}$ equals

(a) $\sec x$ (b) $-\sec x$ (c) $\operatorname{cosec} x$ (d) None of these

32. If the length of the subnormal at any point of the curve $y=a^{1-n}x^{n}$ is constant, then n is equal to

(a) 2 (b) 1/2 (c) –1 (d) None of these

33. The minimum value of $27^{\cos x}+81^{\sin x}$ is

(a) $\dfrac{1}{3\sqrt{3}}$ (b) $\dfrac{2}{9\sqrt{3}}$ (c) $\dfrac{1}{9\sqrt{3}}$ (d) None of these

34. $\displaystyle\int_0^{\pi/2}\frac{\tan x}{1+\tan x}dx$ equals

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) None of these

35. The area between the curves $y^2=4x$ and $y=2x$ is

(a) 1/4 unit (b) 1/3 unit (c) 1/2 unit (d) None of these

36. Solution of differential equation

$(1+y^2)\,dx+\left(x-e^{\tan^{-1}y}\right)dy=0$ is

(a) $ye^{\tan^{-1}x}=\tan^{-1}x+c$ (b) $xe^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+c$

(c) $2x=e^{\tan^{-1}y}+c$ (d) None of these

37. If $|\vec a|=10$, $|\vec b|=2$ and $\vec a\cdot\vec b=12$, then $|\vec a\times\vec b|$ equal

(a) 16 (b) 8 (c) 32 (d) None of these

38. If $\vec a,\vec b,\vec c$ are three non-coplanar vectors and $\vec p,\vec q,\vec r$ are vectors defined as follows :

$\vec p=\dfrac{\vec b\times\vec c}{[\vec a\,\vec b\,\vec c]},\ \vec q=\dfrac{\vec c\times\vec a}{[\vec a\,\vec b\,\vec c]},\ \vec r=\dfrac{\vec a\times\vec b}{[\vec a\,\vec b\,\vec c]}$ then

$(\vec a+\vec b).\vec p+(\vec b+\vec c)\,\vec q+(\vec c+\vec a).\,\vec r$ is equal to

(a) 0 (b) 1 (c) 3 (d) None of these

39. For any two events A and B, $P(A+B)$ equals

(a) $P(A)+P(B)$ (b) $P(A)+P(B)+P(AB)$

(c) $P(A)+P(B)-P(AB)$ (d) None o f these

40. $\tan\alpha+2\tan 2\alpha+4\tan 4\alpha+8\cot 8\alpha$ is equal to

(a) 0 (b) $\cot\alpha$ (c) $\tan 16\alpha$ (d) None of these

41. In a triangle ABC, $\dfrac{b-c}{r_1}+\dfrac{c-a}{r_2}+\dfrac{a-b}{r_3}$ is equal to

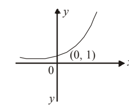
(a) 1 (b) 0 (c) abc (d) None o f these

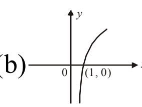
42. Number of real solutions of

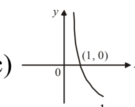
$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\pi/2$ is

(a) 0 (b) 1 (c) 2 (d) None of these

43. Graph of logarithmic function $\log_a x$ $(a>1)$ is

(a) 

(b) 

(c) 

(d)None of these

44. In definite integral as the limit of a sum $\int_0^1 f(x)\,dx$ can be written as

(a) $\displaystyle\operatorname*{Lt}_{x\to 0}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)\cdot\frac{1}{r}$ (b) $\displaystyle\operatorname*{Lt}_{n\to\infty}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)\cdot\frac{1}{n}$

(c) $\displaystyle\operatorname*{Lt}_{n\to\infty}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)\cdot\frac{1}{r}$ (d) None of these

SECTION - C

INTERACTIVE SECTION

COMPREHENSION - 1

Let P,Q,R,S and T are five sets about the quadratic equation $(a-5)x^2-2ax+(a-4)=0$, $a\neq 5$ such that

P : All values of a for which the product of roots of given quadratic equation is positive.

Q : All values of a for which product of roots of given quadratic equation is negative.

R : All values of a for which the product of real roots of given quadratic equation is positive.

S : All values of a for which the roots of given quadratic equation are real.

T : All values of a for which the given quadratic equation has complex roots.

On the basis of above information, answer the following questions:

45. Which statement is correct regarding sets *P, R* and *T*

(a) $P\subseteq R$ (b) $R\subseteq T$ (c) $T\subseteq P$ (d) None of these

where *r'* is the set of real lnumbers.

46. Which statement is correct?

(a) least positive integer for set *R* is 2

(b) least positive integer for set R is 3
(c) greatest positive integer for set T is 3
(d) None of the above

47. If coefficient of x and constant term changes to each other, then
(a) no. of positive integral solutions of a is 4 in P
(b) no. of positive integral solutions of a is 3 in Q
(c) greatest integral value of a is 6 in P
(d) None of these

COMPREHENSION - 2

The locus of a moving point is the path traced out by that point under one or more given conditions. Technically, a locus represents the 'set of all points' which lies on it.
A relation f(x,y) =0 between x and y which is satisfied by each point on the locus and such that each point satisfying the equation is on the locus is called the equation of the locus.

On the basis of above information, answer the following questions :

48. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$, where t is a parameter, is
(a) $(3x-1)^2+(3y)^2=a^2-b^2$ (b) $(3x-1)^2+(3y)^2=a^2+b^2$
(c) $(3x+1)^2+(3y)^2=a^2+b^2$ (d) None of these

49. Let A (2, –3) and B (–2, 1) be vertices of a ΔABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
(a) $2x + 3y = 9$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) None of these

50. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P (h,k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Then the locus of the point P is
(a) $y = 3x-2$ or $y =-3x-2$ (b) $y = 2x-1$ or $y =-2x-1$
(c) $y = 2x+1$ or $y =-2x+1$ (d) None of these

☺ **END OF THE EXAM** ☺

ANSWERS

1. (b)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (a)
6. (a)	**7.** (b)	**8.** (c)	9. (c)	**10.** (c)
11. (a)	**12.** (c)	**13.** (b)	14. (b)	**15.** (d)
16. (c)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (b)
21. (c)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (c)
26. (c)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (b)
36. (b)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (b)
41. (b)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (c)
46. (b)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (c)

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